PROOF. From the center D join DB. The angles at the base AB will be (Eu. I. 5) equal, and likewise at the base BC, in the triangles ADB, CDB. Wherefore in the triangle ABC the two angles at the base AC will be together equal to the whole angle ABC. Therefore the three angles of the triangle ABC will be together equal to, or greater, or less than two right angles according as the angle at the point B was right, or obtuse, or acute. Therefore from any triangle ABC, of which the angle at the point B is inscribed in any semicircle, whose diameter is AC, is established (P.XV) the hypothesis of right angle, or obtuse angle, or acute angle, according as indeed the angle at the point B is right, or obtuse, or acute. Quod erat demonstrandum.

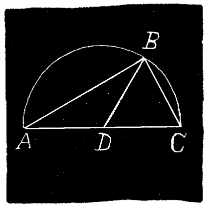


Fig. 17.

[To be continued.]

AN ATTEMPT TO DEMONSTRATE THE 11th AXIOM OF PLAYFAIR'S EUCLID.

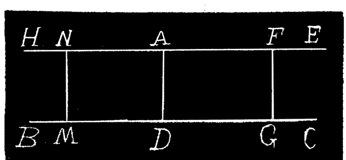
By WARREN HOLDEN, Professor of Mathematics. Girard College, Philadelphia, Pennsylvania.

"*Through a given point one line, and only one, can be drawn parallel to a given line.*"

1st. Two lines perpendicular to a third never intersect, how far soever they be produced. *Halsted's Lobatschewsky's Geometry.* Page 12. Art. 4.

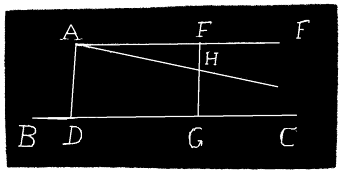
2d. Parallel straight lines are such as are in the same plane, and which, being produced ever so far both ways, do not meet.

3d. Paralles are everywhere equidistant. From A draw AD perpendicular to BC, and through A draw HAE perpendicular to AD. AE and DC being perpendicular to the same line AD are parallel (1st and 2d). From *any* point F let fall the perpendicular FG upon BC. Lay off DM equal to DG, an erect the perpendicular MN. Fold over the part of the figure to the right of AD upon AD as an axis until it falls upon the part to the left. Since A, D, G and M are right angles, and since DM equals DG by construction, AF must fall upon AN, and GF upon MN. The point F is found upon AN and MN, at their intersection N. Therefore GF equals MN. Since F is *any* point, the parallels are everywhere equidistant.



4th. Through a given point one line, and only one, can be drawn parallel

to a given line. Draw AD perpendicular to BC, and through A draw AE perpendicular to AD. Then (1st and 2d) AE and DC are parallel; and the perpendicular FG equals AD, by (3d). Now suppose another line AH parallel to BC. Then HG equals AD or its equal FG. When HG equals FG, AH and AF coincide. Therefore, through a given point one line, and only one, can be drawn parallel to a given line.



The above demonstration may be made without the use of the word parallel. Thus: Through a given point one line, and only one, can be drawn equidistant from a given line.

With the figure drawn as in No. 3, begin the demonstration with the words: From *any* point F let fall the perpendicular &c., to prove the lines equidistant. Then with the same figure as in No. 4, and substituting the word equidistant for parallel, we have the demonstration.

CRADLE-ROCKING BY ELLEPTIC FUNCTIONS.

By F. P. MATZ, M. Sc., Ph. D., New Windsor, Maryland.

After adopting the *gravitation-unit* of force, the equation of motion of the pendulum may be written $(h^2+k^2)(W/g)(d^2\theta/dt^2)=-Wh\sin\theta$....(1). Briefly making $(h+k^2/h)=l$ and $g/l=n^2$, we obtain from (1)

$\frac{1}{2}(d\theta/dt)^2=n^2(\text{vers}\,\alpha-\text{vers}\,\theta)$....(2), in which, according to Sir William Thomson (Lord Kelvin), n is the *angular speed* of the pendulum, Divide *semicircularly* the pendulum-bob, turn downward the convex sides of these divisions centrally joined by a rectilinear axis of inappreciable length, and let the pendulum-rod bisect this rectilinear axis. In the position specified. these divisions constitute the *rockers* of an old-fashioned cradle; and this cradle we regard as placed upon a perfectly rough horizontal plane. Detaching the pendulum rod from the point of suspension, we have to consider *the rocking*, or the rolling oscillations on a horizontal plane, of a material body resting on a semicircular base. Let r=the radius of the equal semicircular rockers. Consider the *line* joining the points of tangency of the rockers with the horizontal plane, as the instantaneous axis of rotation; then, after obvious transformations, (2) becomes $\frac{1}{2}(r^2-2hr\cos\theta+h^2+k^2)(d\theta/dt)^2=gh(\text{vers}\,\alpha-\text{vers}\,\theta)$....(3).